Equal Groups, Arrays, and Multiplication

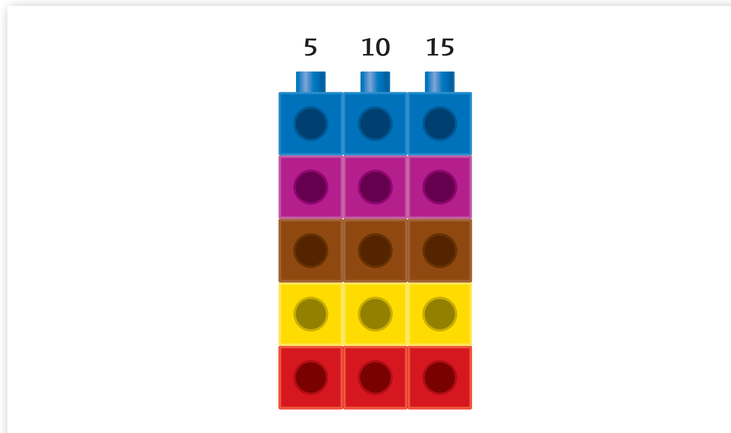
Throughout the year in Grade 2, students work on counting by groups. Although they are not formally using multiplication notation, they are building a foundation of knowledge for multiplication. This unit helps them develop a deeper understanding of accumulation of equal groups. They encounter some of the "counting by" patterns they are coming to know—counting by 2s and by 5s—and they work with less familiar counting sequences, such as counting by 3s, 4s, and 6s.

In this unit, students build and model multiplicative situations that involve accumulating equal groups. Some of the contexts are modeled by arrays; some are not. For example, students build arrays out of connecting cubes to represent buildings, adding on, for instance, three cubes for each "floor" of the building. The number of cubes increases by three each time—3, 6, 9, 12, and so on—as the number of "floors" increases by one. Pattern blocks provide a multiplicative context that does not involve arrays. When they work with pattern blocks, students notice, for example, that it takes two trapezoid pattern blocks to cover each of the hexagons. Each time one more hexagon is covered, the number of trapezoids increases by two.

These contexts give students many opportunities to add or count by equal groups. In Session 2.1, students build arrays to represent buildings with 3 rooms on each floor, discuss how to find the number of rooms in five floors, and then use this information to find the number of rooms in ten floors. During this discussion, you can begin to see the range of ways in which your students think about equal groups.

Some students use the equal number of rooms per floor (the equal number of cubes in each row of the array) to find the total. They add on or count by that number. They might say, "3 plus 3 is 6, and then 3 is 9, and then another 3—10, 11, 12—and then 13, 14, 15." Many students use a strategy like this one, adding for combinations they know, such as 3 + 3, and counting on for less familiar combinations, such as 12 + 3. Some students build on the combinations they know (e.g., "3 plus 3 is 6, and then there's another 3 plus 3 is 6, so that's 6 plus 6 is 12, and then another 3—13, 14, 15").

Because students are more familiar with counting by 5s than with counting by 3s, some students notice that they can count by the columns of the building (the columns of the array), rather than by the layers (rows). Since there are five floors, they count each stack of rooms—5, 10, 15—often running their fingers up the columns to show the 5s.



Counting by the number of cubes in each column is another effective method of using equal groups to help find the total in the cube-building context.

Many students count by ones but use the equal groupings to help them keep track. For example, one student said, "If you count around the top five times, you get to 15." She counted the top row by ones 5 times, grouping the ones by 3s, with a pause in between: 1, 2, 3, . . . 4, 5, 6, . . . 7, 8, 9, . . .10, 11, 12, . . . 13, 14, 15. However, some students will not yet see that noticing equal groups can help them count the total number of cubes. They count by ones without making use of the structure of equal groups. As the unit continues, help these students begin to notice equal groups by encouraging them to combine at least two groups before they count on the rest by ones.

Keep in mind that counting by 3s, 4s, or 6s is not familiar for most second graders. It is not expected that they will be fluent with counting by these numbers by the end of this unit, but they will gain more experience with combinations of 3s, 4s, and 6s. Encourage students to use what they know, such as the doubles 3 + 3 and 4 + 4, and to build on those to make further combinations. For more about how students use doubling to help them find the total number of rooms for ten floors of a cube building, see the **Teacher Note 6:** Using a Doubling Strategy to Reason about Equal Groups.